Learning From Interpretation Transitions with Unknowns

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Outline

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HOMI-LUNG: A strong consortium to fill the gap.
Pneumonia is a heart matter too (and vice-versa).

HOMI-LUNG

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Unraveling Hospital Acquired Pneumonia:

- **Data source**: Hospital-acquired patient data from multiple cohorts.
- **Small sample size, high-dimensional data**: A few hundred samples analyzed from about 5000 features over short-term care (days).
- **Significant data gaps**: Up to 70% unknown values per patient.
- **Primary research goal**: Identify underlying patterns and rules driving Hospital Acquired Pneumonia, uncovering novel insights.

ID	HAP	Sex	Age	Hospital	clinical_1	clinical_2	clinical_3	metabolite_0	metabolite_1	metabolite_2	metabolite_2	metabolite_2	metabolite_2
0	False	M	28	1	High	123	0.123	172	0.1	0.3			0.3
1	True	M	53	2		234		164	0.4	0.8			0.8
2	True	F	24	1	Medium	800	0.5				0.9	0.9	0.9
3	False	F	77	1		321	0.23	156			0.2	0.2	0.2
4	False	F	34	5	Low		0.001	153			0.7	0.7	

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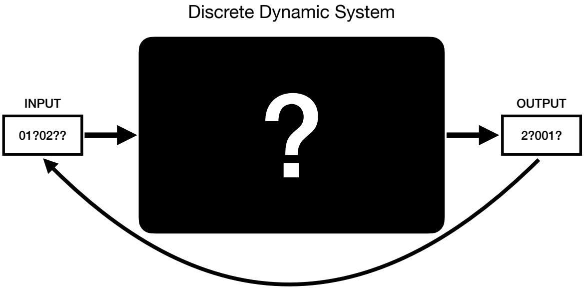
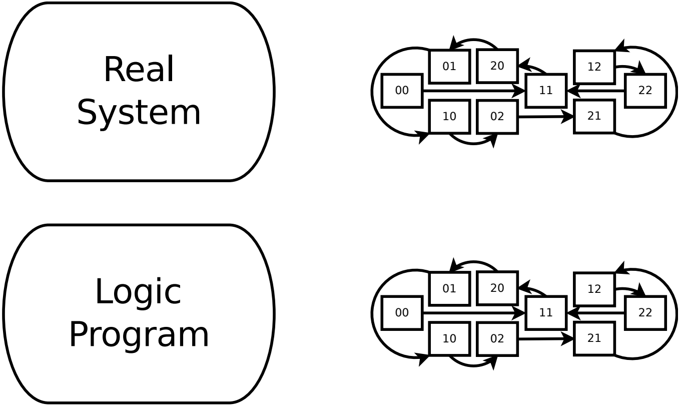
Learning From Interpretation Transitions



- **Idea:**
  Learn black-box **internal mechanics** from its input/output states.
- **Discrete System:**
  Input/output are vectors containing discrete and unknowns values.
- **Dynamic System:**
  System output become the next input (require vectors of same size).

Learning From Interpretation Transitions

Goal: produce a logic program with the same behavior as the system observed.



Logic Rule

$$\underbrace{\mathrm{v}_0^{val_0}}_{\substack{\textit{head}\\ \text{target atom}}} \leftarrow \underbrace{\mathrm{v}_1^{val_1} \wedge \mathrm{v}_2^{val_2} \wedge \ldots \wedge \mathrm{v}_n^{val_n}}_{\substack{\textit{body}\\ \text{feature atoms}}}.$$

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- $\mathrm{v}_0, \mathrm{v}_1, \mathrm{v}_2, \ldots, \mathrm{v}_n$: variables $\quad a_t,\ a_{t-1},\ b_t,\ b_{t-1},\ z_t,\ z_{t-1}$
  - Variables are split into feature ($\mathcal{F}$) and target ($\mathcal{T}$) variables
  - $\mathrm{v}_0 \in \mathcal{T}$ $\quad a_t,\ b_t,\ z_t$
  - $\mathrm{v}_1, \mathrm{v}_2, \ldots, \mathrm{v}_n \in \mathcal{F}$ $\quad a_{t-1},\ b_{t-1},\ z_{t-1}$
  - Implicit time step: t in the *head* and $t-1$ in the *body*

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  - Implicit time step: t in the *head* and $t-1$ in the *body*
- $val_0, val_1, val_2, \ldots, val_n$: values $0, 1, 2, \ldots$
  - $val_i \in \mathrm{dom}(\mathrm{v}_i)$

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- $val_0, val_1, val_2, \ldots, val_n$: values $0, 1, 2, \ldots$
  - $val_i \in \mathrm{dom}(\mathrm{v}_i)$
- All atoms in the *body* are in conjunction
- $\leftarrow$ is the (reverse) implication

Interpretation of a Logic Rule

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Examples:

$$a_t^1 \leftarrow a_{t-1}^2 \wedge b_{t-1}^0 \wedge z_{t-1}^1.$$
$$b_t^1 \leftarrow z_{t-1}^1.$$
$$z_t^0 \leftarrow \top.$$

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A rule R **matches** a state s iff *body* $\subseteq s$

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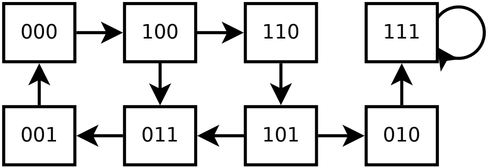
Semantics = From this information, what are the next possible state(s)?
(Similar to discrete networks)

Example: LFIT with complete observations



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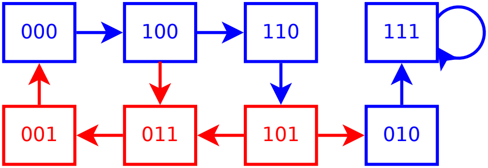

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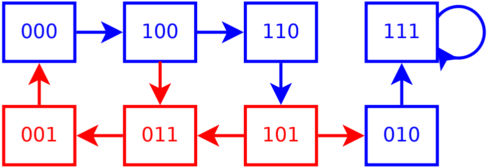

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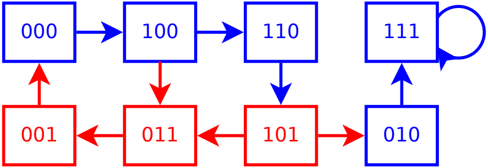

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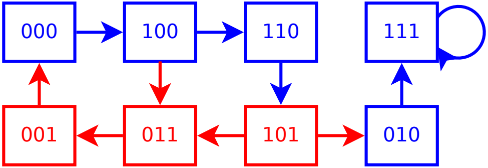

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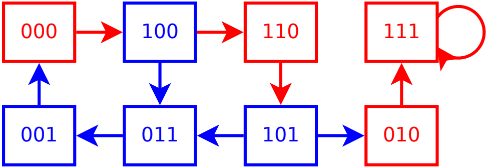

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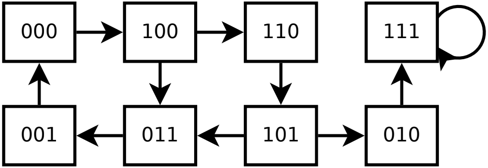

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$a_t^1 \leftarrow c_{t-1}^0$
$a_t^1 \leftarrow a_{t-1}^1, b_{t-1}^1$

$a_t^0 \leftarrow a_{t-1}^0, c_{t-1}^1$
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$b_t^1 \leftarrow a_{t-1}^1, b_{t-1}^0$
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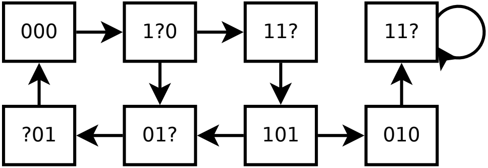
Facing the Unknowns

Goal: Learn a model from incomplete data, acknowledging unknown values to ensure an overapproximation of the dynamics.



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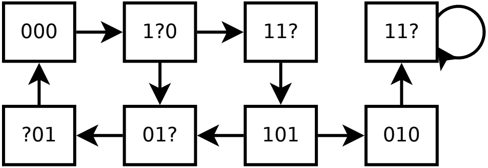

$a^1_t \leftarrow c^0_{t-1}$
$a^1_t \leftarrow a^1_{t-1}, b^1_{t-1}$

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$c^1_t \leftarrow a^1_{t-1}$
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Uncertain Equality

Unknown values can mask differences between states, making it harder to identify examples of target atoms occurring or not.

000 = 000 | 000 ≠ 010 | 000 ≠ ?10

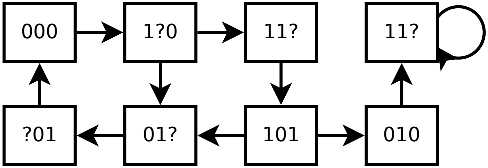
000 =? 0?0 | ?00 =? 0?0

Definition (State uncertain equality)

Two states are uncertain equal, denoted $s \overset{\mathcal{X}}{\backsimeq} s'$ when

$$\forall \mathrm{v}^{val} \in s, \forall \mathrm{w}^{val'} \in s', \mathrm{v} = \mathrm{w} \implies val = val'$$

Example: Usual LFIT case



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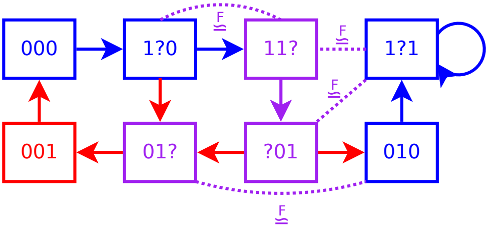

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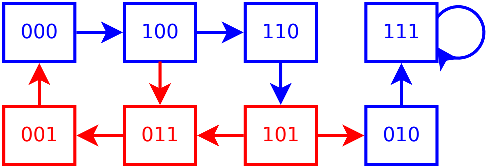

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Conclusions

Contributions

- Extension of **LFIT** to learn from observations with **unknown** values
- Guarantee **over-aproximation** of both **dynamics** and **minimal rules**

Outlooks

- Develop **heuristics** to go beyond theoretical safety
- Adapt and apply the method for **real data** for the Homi-Lung project

Project

Manuscript

Source Code